An Introduction to *Mathematica*

The Basics

The first thing you need to know is that *Mathematica* files are called notebooks and their extensions are .nb. Each notebook consists of cells, of which there are the three types: text, input, and output. Text cells are for text. You're reading one now. You type things in as in any other word processor. If you look in **Format** in the tool bar, you will see that you can type all kinds of neat things, like **bold** or *italic,* large or small, and in color: red, blue, green.

Input cells are the default cell type in *Mathematica*. These are where you do mathematics. To create a new cell, move the cursor down until it becomes a horizontal line, click once, and start typing.

```
This is an input cell.
```

Now we're back to a text cell. If you want a new cell to be text, you could go to **Format** in the tool bar, click on *Style*, and then click on *Text* or you could be sneaky and just hit Alt+7.

There are two ways to tell the difference between text and input cells. One is to look to the right at the brackets on the right side of each page(on the screen they will be blue). Text cells have the two little lines at the top whereas input cells have a diagonal line there.The second is the type face. It will be bold and centered for input cells, as you can see in the first input cell above.

You also need to know that in text and input cells, you can copy, paste, and otherwise edit as you would in any other word processor. This will save you lots of time, and frustration, later, when we're doing some fancy tables or graphs. You see, *Mathematica* is so picky about its syntax. If you're supposed to have a **[** and you put a **{**, it has a fit. Copying and pasting will help eliminate that and give you opportunity to give *Mathematica* something to really have fits about.

Always use parentheses in algebra and arithmetic. The [ ] and { } are reserved for other *Mathematica* commands. The [ ] are for functions and the { } are for lists and intervals.

To have *Mathematica* compute something, you enter it in an input cell and then either hit the shift and enter keys simultaneously on the keyboard or the enter key on the number pad. This will give you the third type of cell, output. You will be able to distinguish between input and output cells since *Mathematica* labels them as **In[ ]** and **Out[ ]**. When you first hit the shift+enter to compute something, it will take a few extra seconds. *Mathematica* has to first "load the kernel." The kernel is where *Mathematica* does its calculations. This is all you need to know, for now.

Mathematica follows the usual algebraic order of operations: parentheses first, then exponentiation, multiplication/division, and finally addition/subtraction. Here is your first syntax lesson: ^ is exponentiation(it's above the 6 key), ***** is multiplication(it's above the 8 key), and / is division.

And as with any software package, be sure to save your work often. If you do make an error, *Mathematica* will give you an error message. It will not automatically correct it for you. Syntax and arithmetic errors are in blue and really good errors are in red. Don't worry, you'll get to see plenty of both.

Let's do some arithmetic.What is 6 plus 4 squared divided by 7? You will see that you have to be careful with your ( ). If you want a decimal approximation use **N**[ ]. Note the **[ ]** and not the **( )** or **{ }**. The **%** is a shortcut for having *Mathematica* use the previous output. Notice how the value of **%** changes with each Output.

```
6 + 4^2 / 7
```

$\frac{58}{7}$

```
N[%]
```

8.28571

```
(6 + 4^2) / 7
```

$\frac{22}{7}$

```
N[%]
```

3.14286

```
(6 + 4)^2 / 7
```

$\frac{100}{7}$

```
N[%]
```

14.2857

Now it's algebra time. The three most frequently used commands are **Factor**, **Simplify**, and **Expand**. The capitals are not typos. ALL *Mathematica* commands begin with capitals. There are commands with more than just the first letter capitalized. Also,whatever you want to **Factor**, **Simplify**, or **Expand** must be enclosed in **[ ]**. Again,

Use only ( ) for algebra and arithmetic. Do not use the [ ] and { }.

Here are some examples. You may notice that *Mathematica* will write sometimes 1+x instead of x+1, like we usually do. That's fine. You could have it write things the other way by going to **Cell** in the toolbar, going down to *Default Output Format Type*, and then going to *TraditionalForm,* like we have done here. The first example factors $x^2 + 6x + 5$. The second simplifies the fraction $\frac{x^2 y^4}{x^4 y^2}$. And the third multiplies out $(x^2 + 6x - 2)(x^2 - 3x + 4)$.

```
Factor[x^2 + 6 x + 5]
```

$(x+1)(x+5)$

```
Simplify[(x^2 y^4) / (x^4 y^2)]
```

$\frac{y^2}{x^2}$

```
Expand[(x^2 + 6 x - 2) (x^2 - 3 x + 4)]
```

$x^4 + 3x^3 - 16x^2 + 30x - 8$

Mathematica has built-in functions. It has all the trig functions as well as exponents and logs. As with the other commands, they all begin with capitals. The main ones are given as examples below. The syntax for π is **Pi,** e is **E,** and ln is **Log.** If you want a logarithm with another base b, use **Log[b, x]**. This is $\log_b x$. *Mathematica* assumes that all angles are in radians. Note this with the first two examples.

```
Sin[Pi / 6]
```

$\frac{1}{2}$

```
Sin[30]
```

sin(30)

```
N[%]
```

−0.988032

```
Cos[Pi / 6]
```

$\frac{\sqrt{3}}{2}$

```
Tan[Pi / 6]
```

$\frac{1}{\sqrt{3}}$

```
Sec[Pi / 6]
```

$\frac{2}{\sqrt{3}}$

```
N[E]
```

2.71828

```
E^2
```

e^2

```
N[%]
```

7.38906

```
Log[E^2]
```

2

```
Log[9, 3]
```

$\frac{1}{2}$

```
Log[3, 9]
```

2

Can you guess what the following does?

```
Sqrt[25]
```

5

It can't be emphasized enough that there are differences in using ( ), [ ], and { }. Always use ( ) in algebra and arithmetic. The [ ] are for functions and the { } are for lists and intervals.

One of the best features of *Mathematica* is that you can define your own functions. The first and most important thing you must remember about defining your own functions is the syntax. While you've been writing f(x) for the f as a function of x ever since you've first learned about them, in *Mathematica,* you enter **f[x_]:=** to define f as a function of x, . Note the **[ ]**, the underscore, and the colon. If you type in **f(x)**, *Mathematica* will read it as "f times x." After you have defined your function, you then refer to it as **f[x]** and you can do f of whatever you want. As a general rule, use lower case letters for your own functions and variables and be aware that some letters, such as the **E**, are syntax. You can name a function almost anything you want. And x is not the only variable in the world.

Consider the function f(x) = x^3 - 2x^2 + 3x - 1. You enter

```
f[x_] := x^3 - 2 x^2 + 3 x - 1
```

To be sure you have entered the function correctly, enter

```
f[x]
```

$x^3 - 2x^2 + 3x - 1$

Mathematica will use the function as <u>you've defined it</u>. It only knows what you tell it. It's your responsibility to make sure you have the right one.

Here are some sample computations.

```
f[2]
```

5

```
f[5.6]
```

128.696

```
f[t]
```

$t^3 - 2t^2 + 3t - 1$

```
f[a + h]
```

$(a+h)^3 - 2(a+h)^2 + 3(a+h) - 1$

```
Expand[%]
```

$a^3 + 3ha^2 - 2a^2 + 3h^2 a - 4ha + 3a + h^3 - 2h^2 + 3h - 1$

Here is a nice example: Simplify $\frac{f(a+h)-f(a)}{h}$. Note that we use **( )** to group the terms of the denominator, but use **[ ]** with the function.

```
Simplify[(f[a + h] - f[a]) / h]
```

$3a^2 + (3h - 4)a + h^2 - 2h + 3$

To denote multiplication, you can either use the *****, **( )**, or a blank space. We've already seen an example using the **( )**. Here are examples of the other two:

```
circumference[r_] := 2 Pi r
area[r_] := Pi * r^2
```

```
circumference[5]
```

10π

```
area[5]
```

25π

Look at what would happen if you didn't use the ***** or a space. *Mathematica* considers the **Pir** to be one variable. So be careful.

```
2 Pir
```

2 Pir

```
Pir^2
```

Pir^2

After having defined several functions and used several letters, you may want to start over. Use the **Clear** command. *Mathematica* remembers everything. If you have defined two functions using **f,** *Mathematica* may not use the one you want. If you want to use the new function f(x) = sin πx, you first clear the old f and then define the new one.

```
Clear[f]
```

```
f[x_] := Sin[Pi * x]
```

```
f[2]
```

0

The first f(2) was 5, so we have the right f. If you're not careful, however, you could end up using the wrong function.

Of course, you'll hardly ever finish a lab in one sitting. So you'll save your work on a diskette to use later. When you do, you'll have to redefine your functions and such. *Mathematica* won't remember what you did before. Before starting any new session, you may first want to clear the variable and function names you'll be using.

Mathematica can re-open your files, but it won't know anything until you tell it. You don't have to retype anything. Just put the cursor on what you want to redefine and hit shift+enter. Save your work often.

Consdier a more complicated function, like one you'll be working with later on: Let f(x)= $\frac{\sqrt{x}-3}{x-9}$.

```
Clear[f, x]
```

```
f[x_] := Sqrt[x] - 3 / x - 9
```

Is this the right f(x)? Check it.

```
f[x]
```

$\sqrt{x} - 9 - \frac{3}{x}$

Not quite. What happened? There are no **( )**. The $\sqrt{x}\,-\,3$ is the numerator of the fraction and you want to divide it by the denominator $x-9$. Here is the proper way to define f(x). You must group the top and bottom with **( )**. Please be aware of this.

```
Clear[f]
```

```
f[x_] := (Sqrt[x] - 3) / (x - 9)
```

```
f[x]
```

$\frac{\sqrt{x}-3}{x-9}$

Finally you need to learn how to define piecewise functions. Let g(x) = $\begin{cases} x^2+1 \text{ if } x \le 2 \\ 2x+1 \text{ if } x > 2 \end{cases}$. It will look as though you've defined g twice. In a sense you have: you define it for x≤2 and then for x>2. To get the **≤**, enter <= and *Mathematica* will automatically convert it. Note that you can define g in one input cell. To see if you defined it correctly, check g at some values you know, such as x=2 and x=3. You can see that g(2)=5 and g(3)=7. You can do g of whatever, just as you would with other functions.

```
g[x_] := x^2 + 1 /; x ≤ 2
g[x_] := 2 x + 1 /; x > 2
```

```
g[2]
```

5

```
g[3]
```

7

Practice #1

0. Do all of these exercises in a new notebook. Also, be aware that you will have to remember some mathematics while doing these.

1. Find exact and approximate values of the following: a.) $2+3/4^2$
   b.) $(2+3)/4^2$
   c.) $2+(3/4)^2$
   d.) $(2+3/4)^2$

2. Define the following functions in *Mathematica:*
   a.) $f(x)=\frac{\sqrt{x+1}+x}{x^2+1}$ b.) $g(t) = \sin(\pi/t)$ c.) $h(x) = \begin{cases} 1-x^2 \text{ if } x < 1 \\ 4x-1 \text{ if } x \geq 1 \end{cases}$.
   Check to be sure that you have entered them correctly.

3. Use the above functions to compute the following:
   a.) f(3) b.) f(7.8) c.) $f(-\pi)$

   d.) g(3) e.) g(3.1) f.) g(-5.7)

   g.) h(1) h.) h(1.1) i.) h(-3)+h(3)

4. Factor $6x^4 + 13x^3 - 18x^2 - 7x + 6$. Check *Mathematica* by multiplying out its answer.

5. Simplify $\frac{f(x)-f(2)}{x-2}$ for $f(x) = 3x^4 - 2x^2 + x - 1$.

6. Now make your work look nice. Go to the *Style* part of the **Format** in the tool bar.
   You'll see lots of neat things you can do. The large Practice #1 was done using *Title*. Put in *Subtitle*s for each of the above problems. Also in **Format**, toward the bottom, is *Text Alignment* features. Left justify your subtitles.

Tables and Graphs

Tables and graphs will be the two features of *Mathematica* you will use most. You will need to create tables of values of certain functions around a given point. *Mathematica* will be able to calculate and organize them much better than you could, even with a calculator. It's always good to see the graph of a function. The following will give the table of values for the function f(x)=x^2 for x=1,2,3,4,5. Some variable names are cleared first.

```
Clear[f, g, h, x, t]
```

```
f[x_] := x^2
```

```
TableForm[Table[{x, f[x]}, {x, 1, 5, 1}], TableHeadings → {None, {"x", "f(x)"}}]
```

x	f(x)
1	1
2	4
3	9
4	16
5	25

The table is created with the **Table[{x,f[x]},{x,1,5,1}]** part. Remember, the **{ }** are for lists. We want the table to contain the x-values with the corresponding f(x)-values. Then we want x to start at 1 and go to 5 in steps of 1. That's the **{x,1,5,1}**. The step size is the difference between successive x values. You may call this an increment or denote it by Δx. For example, the step size for the list {0,.5,1,1.5,2} is .5. This would be entered as {x,0,2,.5}. The **TableForm[ ]** part is how we make it look nice. The "arrow" after **TableHeadings** is formed by the minus (-) and greater-than (>) keys. You enter - > and *Mathematica* will automatically convert it. Just put the **None** there for now. As for the actual headings, you can have anything you want, but each one must be in quotes and make sure that you have one heading for each column. Carefully enter the above. Once you get it right, you can then copy and paste.

This next example will be quite useful for your first lab. You want to examine the values of the function g(x) = $\frac{\sin x}{x}$ for x near 0. Here are two tables. The values of x for the first one start at -1, end at 1, and have a step size of .25. These values are -1, -.75, -.5, -.25, ...

```
g[x_] := Sin[x] / x
```

```
g[x]
```

$$\frac{\sin(x)}{x}$$

```
TableForm[Table[{x, g[x]}, {x, -1, 1, .25}],
 TableHeadings → {None, {"x-values near 0", "corresponding values of g"}}]
```

Power::infy : Infinite expression $\frac{1}{0.}$ encountered.

∞::indet : Indeterminate expression 0. ComplexInfinity encountered.

x−values near 0	corresponding values of g
−1	sin(1)
−0.75	0.908852
−0.5	0.958851
−0.25	0.989616
0.	Indeterminate
0.25	0.989616
0.5	0.958851
0.75	0.908852
1.	0.841471

Note the error message, which would be in blue on the screen. Also, in the table itself there is this "Indeterminate." This comes from the value of g at x=0. We know that g(0) is undefined. The second table gets "closer." It goes from x = -.25 to x = .25 in steps of .05.

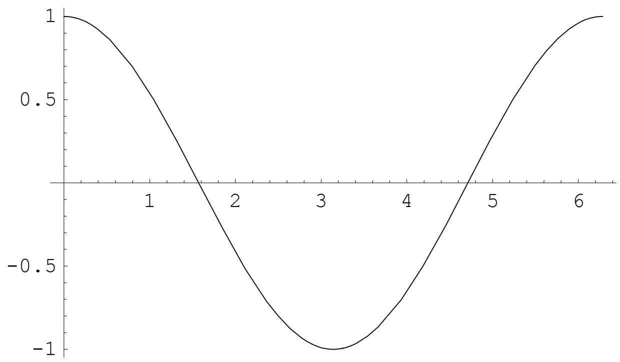
```
TableForm[Table[{x, g[x]}, {x, -.25, .25, .05}],
 TableHeadings -> {None, {"x-values near 0", "corresponding values of g"}}]
```

x−values near 0	corresponding values of g
−0.25	0.989616
−0.2	0.993347
−0.15	0.996254
−0.1	0.998334
−0.05	0.999583
1.38778×10^{-17}	1.
0.05	0.999583
0.1	0.998334
0.15	0.996254
0.2	0.993347
0.25	0.989616

Sometimes *Mathematica* rounds things that it shouldn't. In this case it's g(0). You know that g(0) is undefined, yet due to some rounding errors, *Mathematica* is giving 1. Usually for something like this, you'll get a blue error message like above. The 1.38778×10^{-17} should be 0 and the 1 should be "Indeterminate."

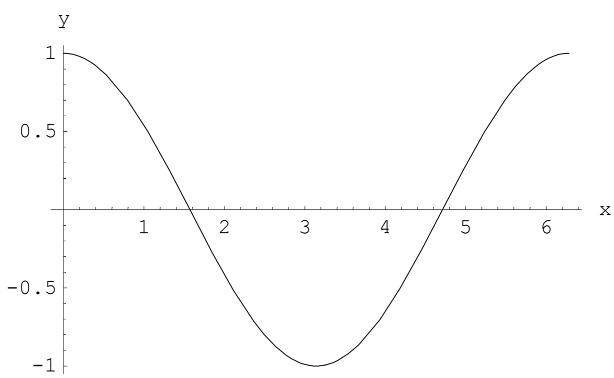
Now it's time to graph. Here's a quick plot of y=cos x on the interval [0,2π].

```
Plot[Cos[x], {x, 0, 2 Pi}];
```



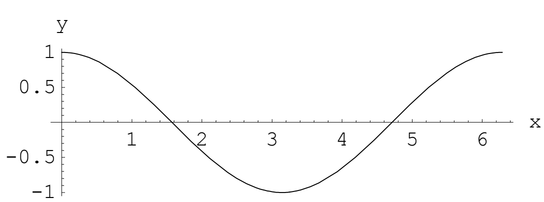
Cosine is a function, so you need the capital **C** and the **[ ]**. *Mathematica* reads the interval as a list. That's the **{x,0,2Pi}**. Now let's make these plots look nicer. We need to label the axes. The semi-colon at the end suppresses an output line that would read -Graphics-.

```
Plot[Cos[x], {x, 0, 2 Pi}, AxesLabel → {"x", "y"}];
```



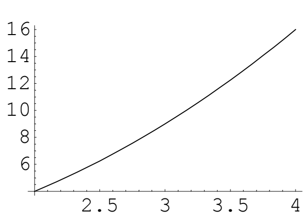
You might have noticed that the axes have different scales. Here's how you fix that.

```
Plot[Cos[x], {x, 0, 2 Pi}, AxesLabel → {"x", "y"}, AspectRatio → Automatic];
```



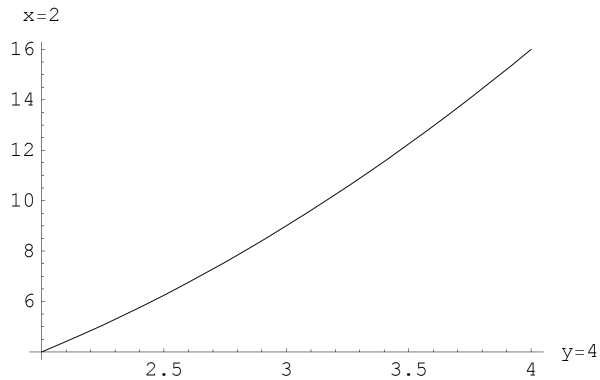
Sometimes the axes you see are not the x and y. An example is the graph of the function $y = x^2$ on the interval [2,4].

```
Plot[x^2, {x, 2, 4}];
```



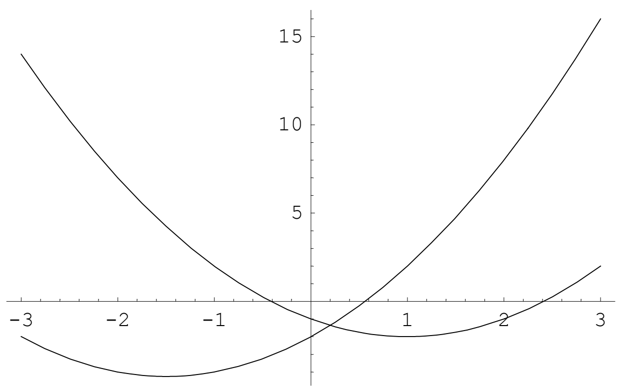
Notice that the axes aren't the x and y. The vertical axis is the line x=2 and the horizontal is the line y=4. Read the scales on each axis to see this. Here is the graph with the axes labeled. To be sure of which axes you have, plot the graph first to see and then plot it again. You can have *Mathematica* show the usual x- and y-axes, but that's for another time.

```
Plot[x^2, {x, 2, 4}, AxesLabel → {"y=4", "x=2"}];
```



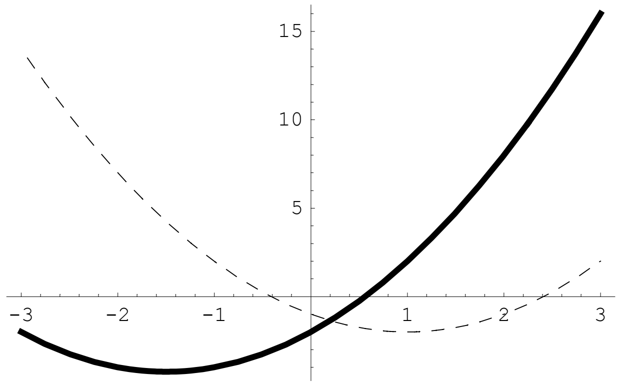
You can also plot more than one function at a time. *Mathematica* has several ways to do this. Here is the easiest. Let's say we want to plot the graphs of $f(x) = x^2 + 3x - 2$ and $g(x) = x^2 - 2x - 1$ on the same axes. We enter the functions in **Plot**[ ] as a list, that is, in **{ }**.

```
Clear[f, g]

f[x_] := x^2 + 3 x - 2
g[x_] := x^2 - 2 x - 1

Plot[{f[x], g[x]}, {x, -3, 3}];
```



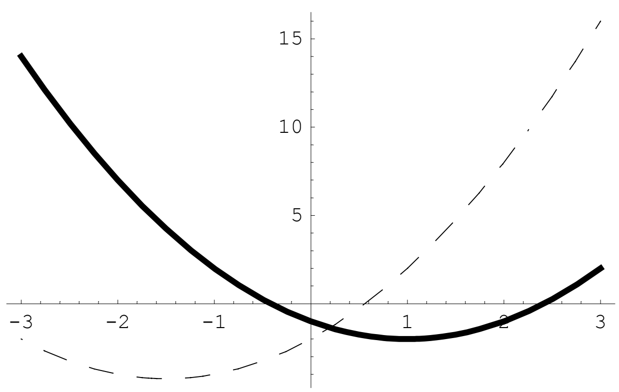
Here's a good question: Which graph is which? Don't worry. *Mathematica* has a few ways to distinguish graphs. Here are two:

```
Plot[{f[x], g[x]}, {x, -3, 3}, PlotStyle → {Thickness[.01], Dashing[{.02}]}];
```



We made the graph of f(x) a bit thicker with the **Thickness[.01]** and we've dashed the graph of g(x) with the **Dashing[{.02}]**. The numbers for these are always between 0 and 1. The larger the number for **Thickness**, the thicker the graph will be. The larger the number for **Dashing**, the longer the dashes will be. If you want the graph of f(x) to be dashed, just put **Dashing** first in the list.

```
Plot[{f[x], g[x]}, {x, -3, 3}, PlotStyle → {Dashing[{.04}], Thickness[.01]}];
```



If you're not sure of what to do, you can always go to the *Help* section. Then click on *Help Browser*. You should be able to find whatever you want there.

Practice #2

0. Do these exercises in a new notebook and remember what you did in Practice #1. Use copy and paste to minimize typing and typos.

1. Let f(x)= $\frac{\sqrt{x+1}-2}{x-3}$. Check that you defined f(x) correctly. Make tables of values for f as x goes from a.) 8 to 10 in steps of .2 b.) 8.8 to 9.2 in steps of .05 c.) 8.95 to 9.05 in steps of .005

2. Plot the graph of g(x) =$x^3 + x^2$ -3x +3 on each of the following intervals: [-2,2], [-1,1], and [-.5,.5]. Label the axes.

3. Plot the graphs of f(x) = cos 2x and g(x) = sin 3x on the interval [0,2π]. Make the graph of f(x) dashed and the graph of g(x) a bit thicker. Label the axes as well.

4. In this problem you will graph the piecewise function h(x) = $\begin{cases} 1-x^2 \text{ if } x \leq 1 \\ 4x-1 \text{ if } x > 1 \end{cases}$. First define h(x).

   Then plot it as you normally would on the interval [-2,3]. Do you see a vertical line in the graph? Is this OK? Explain, in a text cell, why the graph of a function of x CANNOT HAVE a vertical line in it. Then enter the following EXACTLY to graph the function without the vertical line. Don't worry about what the **DisplayFunction** and **Identity** mean.

```
plot1 = Plot[h[x], {x, -2, 1}, DisplayFunction → Identity];
plot2 = Plot[h[x], {x, 1, 3}, DisplayFunction → Identity];
Show[plot1, plot2, DisplayFunction → $DisplayFunction, AxesLabel → {"x", "y=h(x)"}];
```

5. *Mathematica* can also plot graphs in different colors. Find out how to do this by going to the *Help* section, clicking on *Help Browser*, and entering RGBColor. Plot the graphs of the functions in Problem 3 using different colors. Don't print out these graphs, but do write, in a text cell, which colors you used.

6. Now make your work look nice, as you did in **Practice #1**.